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CONDITIONALLY DISTRIBUTION-FREE TEST FOR CENSORED BIVARIATE OBS--ETC(U)
JUL 79 W J PADGETT, L J WEI F49620-79-C-0140

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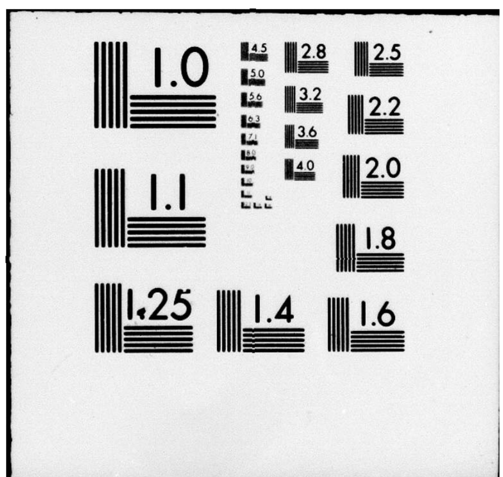

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CONDITIONALLY DISTRIBUTION-FREE TEST FOR CENSORED BIVARIATE OBSERVATIONS*

by

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Statistics Technical Report No. 41
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July, 1979

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A conditionally distribution-free test is proposed for testing the symmetry of a bivariate distribution function with observations which are subject to arbitrary right censorship. In a numerical study, this new test is shown to be more powerful than the sign test under Marshall and Olkin's (1967) bivariate exponential model.

Key words and phrases: Right censorship; Marshall and Olkin bivariate exponential model; permutation test.

1. INTRODUCTION

In many situations, the comparisons between two treatments based on paired observations which are censored in one or both variates may arise. For example, Hammond (1964) has used a matched pair analysis to study smoking in relation to mortality in the United States. Batchelor and Hackett (1970) gave a comparison of the survival times between HL-A closely matched and poorly matched skin allografts on the same badly burned patients. Also, in life testing it may be desirable to compare the life times of two components in a system.

Suppose that

$$(X_1^0,Y_1^0)' , (X_2^0,Y_2^0)' , \cdots , (X_n^0,Y_n^0)' \tag{1.1}$$

are independent, identically distributed random vectors, having $H^0(s,t)$ as their distribution function (d.f.) and having $F^0(s)$ and $G^0(t)$ as their marginal d.f.'s, respectively, where ' denotes vector transpose. The null hypothesis, which is to be tested is

$$H_o: \; H^0(s,t) = H^0(t,s) , \quad \text{for} \quad (s,t)' \in R^2 .$$

Since X_i^0 and Y_j^0 may be censored from the right by variables U_i and V_j, respectively, (1.1) cannot always be observed. The observations available to the experimenter actually consist of the minima

$$\begin{aligned} X_1 &= \min(X_1^0,U_1) , \cdots , X_n = \min(X_n^0,U_n) , \\ Y_1 &= \min(Y_1^0,V_1) , \cdots , Y_n = \min(Y_n^0,V_n) , \end{aligned} \tag{1.2}$$

and two random sequences $\{\delta_1, \cdots ,\delta_n\}$ and $\{\varepsilon_1, \cdots ,\varepsilon_n\}$, where

$$\delta_i = \begin{cases} 1, & \text{if } X_i = X_i^0 , \\ 0, & \text{if } X_i < X_i^0 , \end{cases}$$

$$\epsilon_j = \begin{cases} 1, & \text{if } Y_j = Y_j^0 , \\ 0, & \text{if } Y_j < Y_j^0 . \end{cases}$$

For $i \neq j$, the censoring variables U_i and V_j are assumed to be independent random variables with a common d.f. J. To have the same censoring mechanism for both variates is quite common in paired studies. It is also assumed that $(U_i, V_i)'$ and $(X_i^0, Y_i^0)'$ are independent, $i = 1,2,\cdots,n$.

In the parametric case, the procedure for testing H_o is rather complicated and no useful results have been obtained. However, Holt and Prentice (1974) used the proportional hazards model (Cox (1972)) to analyze the data by Batchelor and Hackett (1970), and Wei (1979) proposed an asymptotically distribution-free test for H_o based on paired observations which are subject to arbitrary right censorship. Since the sample size in paired studies is frequently small, a distribution-free test is highly desirable. Although the sign test is a conditionally distribution-free test for testing H_o, it is rather inefficient when there are too many censored pairs in the data. As an extreme case, for the data $(3^+,4)'$, $(6,5^+)$, $(2^+,4)$, $(9,7^+)$, $(8^+,6^+)$, where "+" denotes censoring, the sign test leads to no conclusion about the null hypothesis H_o.

In this article, a conditionally distribution-free test for H_o is presented in Section 2. In a numerical study, it is shown that the new test

is more powerful than the sign test under Marshall and Olkin's bivariate exponential model (Marshall and Olkin (1967)).

We note that all the results of this article can be easily extended to the case of arbitrarily restricted observations (Mantel (1967)).

2. THE TEST STATISTIC

Let $Z_i = X_i$ and $Z_{n+i} = Y_i$, $i = 1, \cdots, n$. We will say that Z_i is definitely greater than Z_j if $Z_i > Z_j$ and Z_j is observed, and Z_i is definitely less than Z_j if $Z_i < Z_j$ and Z_i is observed. Now, let $\xi_i(\eta_i)$, $i = 1,2, \cdots, n$, be the number of the remaining $(2n-1)$ Z's than which $X_i(Y_i)$ is definitely greater minus the number than which it is definitely less. The original observations (1.2) are then replaced by $(\xi_1,\eta_1)'$, $\cdots$, $(\xi_n,\eta_n)'$. Under H_o and the assumption of an equal censoring mechanism for both variates, all the arrangements of the form

$$(R_1,R_2)', \cdots, (R_{2n-1},R_{2n})'$$

are equally likely, where $(R_{2i-1},R_{2i}) = (\xi_i,\eta_i)$ or (η_i,ξ_i) , $i = 1, \cdots, n$.

The statistic proposed here for testing H_o is

$$W = \sum_{i=1}^{n} R_{2i-1} .$$

Small or large values of $\sum_{i=1}^{n} \xi_i$ lead to the rejection of H_o . Note that $\sum_{i=1}^{n} \xi_i$ is Gehan's (1965) two-sample statistic. Mantel (1967) gave a simple routine to calculate ξ_i and η_i . It should also be noted that scores other than Gehan's (ξ_i,η_i) can be utilized.

The drawback to any permutation test such as W is the usually long and tedious calculations required when the sample size n is large. Fortunately, an asymptotically distribution-free test is available for large sample cases (Wei (1979)). In the rest of this article, we concentrate on the small-sample performance of the W test.

3. THE POWER STUDY

In this section, we study a special alternative hypothesis H_1 : $F^0(s) \le G^0(s)$ for all s and $F^0(s') < G^0(s')$ for some s' , and compare the W test with the sign test under Marshall and Olkin's bivariate exponential model. The survival function of this model with parameters λ_1, λ_2, and λ_{12} can be written as:

$$P\{X^0 \ge s, Y^0 \ge t\} = \exp[-\lambda_1 s - \lambda_2 t - \lambda_{12} \max(s,t)] . \tag{3.1}$$

The two marginal means are

$$EX^0 = 1/(\lambda_1 + \lambda_{12}) \text{ and } EY^0 = 1/(\lambda_2 + \lambda_{12}) .$$

Under this model, the hypotheses to be tested become $H_o : \lambda_1 = \lambda_2$ against $H_1 : \lambda_1 < \lambda_2$.

Three censoring schemes are considered in this comparison:

(A) J(s) is a uniform distribution over $(0, EX^0)$;

(B) J(s) is a uniform distribution over $(0, 2EX^0)$; and

(C) J(s) is a uniform distribution over $(0, 4EX^0)$.

In this numerical study, the censoring variables U_i and V_i are assumed to be independent.

For the sign test and the W test, the proportions of times in the 1,000 Monte Carlo samples generated that H_o was rejected at the $\alpha = .05$ level were calculated for samples of sizes $n = 10$ and 15 from (3.1) with various values of λ_1, λ_2, and λ_{12} . Tables 1 and 2 give the results. As we expected, the W test is uniformly more powerful than the sign test. In addition to the drawback which was illustrated by an example in Section 1, another disadvantage of the sign test is the actual probability of Type I error is far below the specified α value. For example, when $n = 10$, under the severe censorship (A), the empirical levels of the sign test are only .01 as compared with the nominal value $\alpha = .05$.

Table 1. Proportion of times in 1,000 samples of size $n = 10$ that H_o was rejected at $\alpha = .05$ by sign test and W test.

			Censoring Scheme					
			(A)		(B)		(C)	
λ_{12}	λ_1	λ_2	W	Sign	W	Sign	W	Sign
.1	.1	1.0	.85	.60	.91	.73	.92	.76
	.2		.58	.34	.69	.45	.74	.49
	.3		.38	.17	.46	.24	.51	.29
	.4		.25	.09	.32	.14	.36	.18
	.5		.21	.06	.25	.11	.26	.14
	.6		.15	.05	.18	.07	.19	.09
	.7		.09	.02	.11	.05	.13	.06
	.8		.06	.02	.07	.03	.08	.04
	.9		.05	.02	.06	.02	.05	.03
	1.0		.04	.01	.04	.02	.04	.03
.5	.1	1.0	.44	.16	.57	.34	.64	.47
	.2		.33	.09	.42	.20	.48	.30
	.3		.23	.06	.28	.13	.34	.20
	.4		.18	.04	.21	.09	.25	.13
	.5		.15	.02	.18	.06	.19	.09
	.6		.12	.01	.13	.04	.14	.05
	.7		.07	.01	.09	.03	.10	.04
	.8		.06	.01	.07	.02	.07	.03
	.9		.06	.01	.05	.02	.06	.02
	1.0		.04	.01	.05	.01	.04	.02

Table 2. Proportion of times in 1,000 samples of size $n = 15$ that H_o was rejected at $\alpha = .05$ by sign test and W test.

			Censoring Scheme					
			(A)		(B)		(C)	
λ_{12}	λ_1	λ_2	W	Sign	W	Sign	W	Sign
.1	.2	1.0	.79	.55	.86	.66	.88	.74
	.4		.38	.18	.47	.25	.51	.32
	.6		.17	.06	.21	.10	.23	.13
	.8		.09	.03	.10	.04	.11	.05
	1.0		.04	.01	.04	.02	.04	.02
.5	.2	1.0	.48	.25	.60	.42	.67	.50
	.4		.27	.10	.32	.17	.37	.22
	.6		.13	.04	.16	.06	.19	.09
	.8		.07	.01	.09	.03	.09	.04
	1.0		.05	.01	.05	.02	.05	.03

REFERENCES

Batchelor, J. R. and Hackett, M. (1970). HL-A matching in treatment of burned patients with skin allografts. Lancet 2, 581-3.

Cox, D. R. (1972). Regression models and life tables (with Discussion). J. R. Statist. Soc. 34B, 187-220.

Gehan, E. (1965). A generalized Wilcoxon test for comparing arbitrarily single censored samples. Biometrika 52, 203-23.

Hammond, E. C. (1964). Smoking in relation to mortality and morbidity. Findings in first thirty-four months of follow-up in a prospective study started in 1959. J. Nat. Cancer Inst. 32, 1161-88.

Holt, J. D. and Prentice, R. L. (1974). Survival analyses in twin studies and matched pair experiments. Biometrika 61, 17-30.

Mantel, N. (1967). Ranking procedures for arbitrarily restricted observations. Biometrics 23, 65-78.

Marshall, A. W. and Olkin, I. (1967). A multivariate exponential distribution. J. Am. Statist. Assoc. 62, 30-44.

Wei, L. J. (1979). A generalized Gehan and Gilbert test for paired observations which are subject to arbitrary right censorship. T.R. Department of Math., Computer Science, and Statistics, University of South Carolina.